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# Method for Assessing the Electric Power System Reliability of Multiple-Engined Aircraft

Hsing-Juin Lee*
*National Chung-Hsing University, Taiwan, Republic of China*
and
Hsing-Wei Lee†
*Chung-Cheng Institute of Technology, Taiwan, Republic of China*

## Introduction

THE reliability of an electricity generating system is an essential issue in the design analysis of most aircrafts.[1,2] Adequate electricity power supply is critical for successful operations of airborne engines, control surfaces, radar, weaponry, instruments, etc. In an attempt to achieve a high level of reliability for this important airborne electric power system, two or more electric generators are usually arranged in parallel to be driven by each engine of an aircraft as shown in the example for a twin-engine aircraft (Fig. 1), where each electricity generator has a power capacity of 30 kVA. Assuming that with a minimum electric power supply of 30 kVA the aircraft can barely maintain flight in an emergency operation (case A). On the other hand, if the electricity generators together can supply a minimum of 60 kVA, then the aircraft can operate as normal (case B).

To assess the system reliability[3,4] of such a typical aircraft, electricity generating system is important in the design analysis phase of aircraft. For a standard flight session if each electricity generator has a reliability rated at $L_{Gi}$ (where the subscript $i$ represents the designated generator number), then the corresponding failure rate $F_{Gi}$ is equal to $(1\text{-}L_{Gi})$. The reliability and failure rate for each engine are denoted as $L_{Ei}$ and $F_{Ei}$, respectively. Consequently, the probability of success for emergency electricity supply (case A) can be easily expressed as

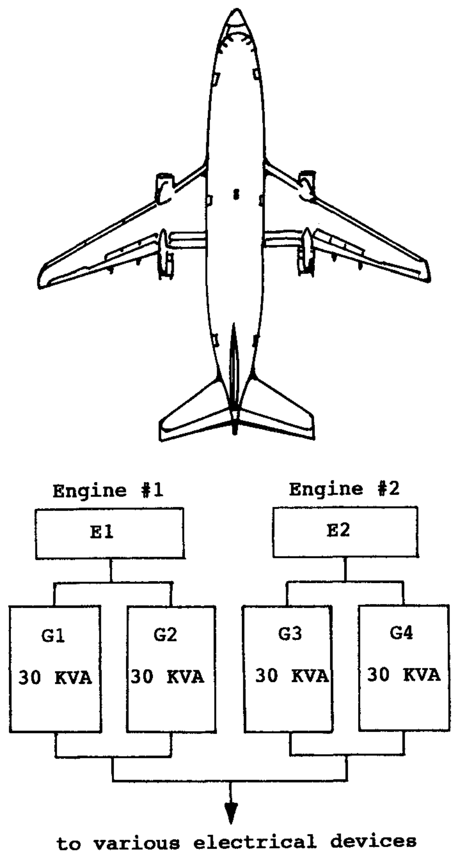


**Fig. 1 Twin-engine four-generator system.**

$$[(P_{\text{success}})_{\text{syst}}]_{30\,\text{kVA}} = 1 - \{1 - [1 - (F_{G1})(F_{G2})]L_{E1}\} \cdot \{1 - [1 - (F_{G3})(F_{G4})]L_{E2}\} \tag{1}$$

But the logical reasoning to obtain the expression of system reliability for the normal power supply (case B) is rather complicated, usually the traditional method with simplex conditional probability concept is employed to attack this problem. In that case, the system reliability can eventually be expressed as

$$[(P_{\text{success}})_{\text{syst}}]_{60\,\text{kVA}} = 1 - \{[((1 - L_{G2})\{1 - [L_{E2}(1 - F_{G3}F_{G4})]\}L_{E1}) + (1 - L_{E2}L_{G3}L_{G4})F_{E1}]L_{G1} + \{[1 - L_{E2}(1 - F_{G3}F_{G4})]L_{E1}L_{G2} + (1 - L_{E2}L_{G3}L_{G4}) \cdot (1 - L_{E1}L_{G2})\}F_{G1}\} \tag{2}$$

The whole reasoning process becomes even more difficult for more complicated cases with more engines and associated electric generators. In order to solve this dilemma, a combinational pivotal decomposition method (CPDM) is developed to assess the system reliability of the aircraft electricity generating system in a much more efficient and systematic manner, both in ideology and implementation, in contrast to the traditional conditional probability methodology. Several typical examples are utilized below to illustrate the necessary definitions, assumptions, concepts, and detailed mathematical procedures.

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*Associate Professor of Mechanical Engineering, Taichung, 40227. Member AIAA.

†Associate Professor of Surveying Engineering and Mapping, Tao Yuan, 33509.

## Conceptually Efficient Combinational Pivotal Decomposition Method

This CPDM method makes use of the pivotal decomposition technique[5] to assess system reliability in an enhanced manner by considering all possible combinational top cases as the pivotal events. Then the conditional reliability of each case can be easily calculated with the aid of binominal distribution. Subsequently, the system reliability is obtained by using the theorem of total probability. This method can be conveniently explained by applying to case B for a twin-engine aircraft. For these two engines there are four mutually exclusive success/failure combinational cases, i.e., $(L_{E1}L_{E2})$, $(L_{E1}F_{E2})$, $(F_{E1}L_{E2})$, and $(F_{E1}F_{E2})$. Then considering each of the above four cases separately, the system reliability for 60 kVA (case B) can be expressed as

$$\begin{aligned}[(P_{\text{success}})_{\text{syst}}]_{60\,\text{kVA}} &= L_{E1}L_{E2}[1 - F_{G1}F_{G2}F_{G3}F_{G4} \\ &\quad - (L_{G1}F_{G2}F_{G3}F_{G4} + F_{G1}L_{G2}F_{G3}F_{G4} + F_{G1}F_{G2}L_{G3}F_{G4} \\ &\quad + F_{G1}F_{G2}F_{G3}L_{G4})] + L_{E1}F_{E2}L_{G1}L_{G2} + F_{E1}L_{E2}L_{G3}L_{G4} \\ &\quad + (F_{E1}F_{E2})0 \end{aligned} \tag{3}$$

Equation (3) shows that under the conditional probability of $(L_{E1}L_{E2})$, which means both engines are in successful condition, then a minimum of any two of the four associated generators should be in good working condition for the aircraft to have sufficient electricity for normal flight operation. Moreover, under the conditional probability of $(L_{E1}F_{E2})$, which means one engine is in good condition and the other engine is in failure condition, then the two generators associated with the sole good engine should all be in working condition to have sufficient electricity supply. The case for $(F_{E1}L_{E2})$ can be calculated in the same manner. On the other hand, while both engines are out of work, there would be no electricity generated by the power system as indicated by the last term in Eq. (3). To avoid overdue mathematical complication and to highlight the CPDM methodology, from now on all the engines are assumed to have the same reliability $L_E$, and all the generators are assumed to have the same reliability $L_G$. Then Eq. (3) can be further simplified as follows:

$$\begin{aligned}[(P_{\text{success}})_{\text{syst}}]_{60\,\text{kVA}} &= L_E L_E[1 - F_G^4 - (4L_G F_G^3)] \\ &\quad + (L_E F_E)L_G^2 + (F_E L_E)L_G^2 + (F_E F_E)0 \end{aligned} \tag{4}$$

Moreover, for a typical two-engine six-generator system, each engine drives three 30-kVA generators in parallel, then the system reliability for a minimum electricity supply of 60 kVA for emergency flight can be expressed as

$$\begin{aligned}[(P_{\text{success}})_{\text{syst}}]_{60\,\text{kVA}} &= L_E L_E(1 - F_G^6 - 6L_G F_G^5) \\ &\quad + (L_E F_E + L_E F_E)(1 - F_G^3 - 3L_G F_G^2) = L_E^2(1 - F_G^6 \\ &\quad - 6L_G F_G^5) + 2L_E F_E(1 - F_G^3 - 3L_G F_G^2) \end{aligned} \tag{5}$$

On the other hand, the system reliability for a minimum electricity supply of 90 kVA for normal flight can be expressed as

$$\begin{aligned}[(P_{\text{success}})_{\text{syst}}]_{90\,\text{kVA}} &= L_E L_E[1 - \tbinom{6}{0}F_G^6 - \tbinom{6}{1}L_G F_G^5 \\ &\quad - \tbinom{6}{2}L_G^2 F_G^4] + L_E F_E L_G^3 + L_E F_E L_G^3 = L_E L_E(1 - F_G^6 \\ &\quad - 6L_G F_G^5 - 15L_G^2 F_G^4) + (L_E F_E + L_E F_E)L_G^3 \\ &\quad = L_E^2(1 - F_G^6 - 6L_G F_G^5 - 15L_G^2 F_G^4) + 2L_E F_E L_G^3 \end{aligned} \tag{6}$$

Where the conventional notation of binomial coefficient $\binom{m}{n}$ is conveniently used throughout the above equation. Furthermore, for a typical four-engine eight-generator system, each engine drives two 30-kVA generators in parallel, then the system reliability for a minimum electricity supply of 60 kVA for emergency flight can be expressed as

$$\begin{aligned}[(P_{\text{success}})_{\text{syst}}]_{60\,\text{kVA}} &= L_E^4[1 - \tbinom{8}{0}F_G^8 - \tbinom{8}{1}L_G F_G^7] \\ &\quad + \tbinom{4}{1}L_E^3 F_E[1 - \tbinom{6}{0}F_G^6 - \tbinom{6}{1}L_G F_G^5] + \tbinom{4}{2}L_E^2 F_E^2 \\ &\quad \cdot[1 - \tbinom{4}{0}F_G^4 - \tbinom{4}{1}L_G F_G^3] + \tbinom{4}{3}L_E F_E^3 L_G^2 = L_E^4(1 \\ &\quad - F_G^8 - 8L_G F_G^7) + 4L_E^3 F_E(1 - F_G^6 - 6L_G F_G^5) \\ &\quad + 6L_E^2 F_E^2(1 - F_G^4 - 4L_G F_G^3) + 4L_E F_E^3 L_G^2 \end{aligned} \tag{7}$$

Likewise, the system reliability for a minimum supply of 90 kVA for normal flight can be expressed as

$$\begin{aligned}[(P_{\text{success}})_{\text{syst}}]_{90\,\text{kVA}} &= L_E^4[1 - \tbinom{8}{0}F_G^8 - \tbinom{8}{1}L_G F_G^7 \\ &\quad - \tbinom{8}{2}L_G^2 F_G^6] + \tbinom{4}{1}L_E^3 F_E[1 - \tbinom{6}{0}F_G^6 - \tbinom{6}{1}F_G^5 L_G \\ &\quad - \tbinom{6}{2}F_G^4 L_G^2] + \tbinom{4}{2}L_E^2 F_E^2[\tbinom{4}{4}L_G^4 - \tbinom{4}{3}L_G^3 F_G] = L_E^4(1 \\ &\quad - F_G^8 - 8L_G F_G^7 - 28L_G^2 F_G^6) + 4L_E^3 F_E(1 - F_G^6 \\ &\quad - 6L_G F_G^5 - 15L_G^2 F_G^4) + 6L_E^2 F_E^2(L_G^4 + 4L_G^3 F_G) \end{aligned} \tag{8}$$

For such relatively complicated engine/generator systems, the combinational pivotal decomposition method has successfully demonstrated its power and efficiency to predict the system reliability in a systematic manner.

## Conclusion

System reliability or survivability of electricity generating is an essential issue in the design analysis of aircraft. In this study, the combinational pivotal decomposition method has been successfully developed and applied to assessing the system reliability of airborne electricity generating system in a much more efficient and systematic manner, both in ideology and implementation, in contrast to the traditional conditional probability methodology. Several typical examples have been utilized for explanation purpose. In fact, the CPDM method cannot only be used to assess the system reliability of airborne electric machines, it can also be used to assess the system combat survivability of two-dimensional truss structures due to gunfire bombardment. Moreover, CPDM may be used to assess the system survivability of space station truss structures due to micrometeoroids/debris impacts, or other causes, such as aging, fatigue, radiation, etc.

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